SUBJECT CODE	SUBJECT	PAPER
C-15-17	**MATHEMATICAL SCIENCES**	**III**

HALL TICKET NUMBER	QUESTION BOOKLET NUMBER
	305148
OMR SHEET NUMBER	

DURATION	MAXIMUM MARKS	NUMBER OF PAGES	NUMBER OF QUESTIONS
2 Hour 30 Minutes	150	16	75

This is to certify that, the entries made in the above portion are correctly written and verified.

Candidate's Signature

Name and Signature of Invigilator

INSTRUCTIONS FOR THE CANDIDATES

1. Write your Hall Ticket Number in the space provided on the top of this page.
2. This paper consists of seventy five multiple-choice type of questions.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested <u>to open the booklet and compulsorily examine it as below :</u>
   (i) To have access to the Question Booklet, tear off the paper seal on the edge of this cover page. Do not accept a booklet without sticker-seal and do not accept an open booklet.
   (ii) **Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faulty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.**
   (iii) After this verification is over, the Test Booklet Number should be entered in the OMR Sheet and the OMR Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example : Ⓐ Ⓑ ● Ⓓ
   where (C) is the correct response.
5. Your responses to the items are to be indicated in the **OMR Answer Sheet given to you.** If you mark at any place other than in the circle in the OMR Answer Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your name or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, you will render yourself liable to disqualification.
9. **The candidate must handover the OMR Answer Sheet to the invigilators at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. The candidate is allowed to take away the carbon copy of OMR Sheet and used Question Paper Booklet at the end of the examination.
10. **Use only Blue/Black Ball point pen.**
11. **Use of any calculator or log table etc., is prohibited.**
12. **There is no negative marks for incorrect answers.**

అభ్యర్థులకు సూచనలు

1. ఈ పుట పై భాగంలో ఇవ్వబడిన స్థలంలో మీ హాల్ టికెట్ నంబరు రాయండి.
2. ఈ ప్రశ్న పత్రము డెబ్బైఐదు బహుళైచ్ఛిక ప్రశ్నలను కలిగి ఉంది.
3. పరీక్ష ప్రారంభమున ఈ ప్రశ్నాపత్రము మీకు ఇవ్వబడుతుంది. మొదటి ఐదు నిమిషములలో ఈ <u>ప్రశ్నాపత్రమును తెరిచి కింద తెలిపిన అంశాలను తప్పనిసరిగా సరిచూసుకోండి.</u>
   (i) ఈ ప్రశ్న పత్రమును చూడడానికి కవర్‌పేజి అంచున ఉన్న కాగితపు సీలును చించండి. స్టిక్కర్ సీలులేని మరియు ఇదివరకే తెరిచి ఉన్న ప్రశ్నాపత్రమును మీరు అంగీకరించవద్దు.
   (ii) కవరు పేజి పై ముద్రించిన సమాచారం ప్రకారం ఈ ప్రశ్నపత్రములోని పేజీల సంఖ్యను మరియు ప్రశ్నల సంఖ్యను సరిచూసుకోండి. పేజీల సంఖ్యకు సంబంధించి గానీ లేదా సూచించిన సంఖ్యలో ప్రశ్నలు లేకపోవుట లేదా నిజప్రతి కాకపోవుట లేదా ప్రశ్నలు క్రమపద్ధతిలో లేకపోవుట లేదా ఏవైనా తేడాలుండుట వంటి దోషపూరితమైన ప్రశ్న పత్రాన్ని వెంటనే మొదటి ఐదు నిమిషాల్లో పరీక్షా పర్యవేక్షకునికి తిరిగి ఇచ్చివేసి దానికి బదులుగా సరిగ్గా ఉన్న ప్రశ్నపత్రాన్ని తీసుకోండి. తదనంతరం ప్రశ్నపత్రము మార్చబడదు అదనపు సమయం ఇవ్వబడదు.
   (iii) పై విధంగా సరిచూసుకొన్న తర్వాత ప్రశ్నాపత్రం సంఖ్యను OMR పత్రము పై అదేవిధంగా OMR పత్రము సంఖ్యను ఈ ప్రశ్నాపత్రము పై నిర్దిష్టస్థలంలో రాయవలెను.
4. ప్రతి ప్రశ్నకు నాలుగు ప్రత్యామ్నాయ ప్రతిస్పందనలు (A), (B), (C) మరియు (D) లుగా ఇవ్వబడ్డాయి. ప్రతి ప్రశ్నకు సరైన ప్రతిస్పందనను ఎన్నుకొని కింద తెలిపిన విధంగా OMR పత్రములో ప్రతి ప్రశ్నా సంఖ్యకు ఇవ్వబడిన నాలుగు వృత్తాల్లో సరైన ప్రతిస్పందనను సూచించే వృత్తాన్ని బాల్ పాయింట్ పెన్‌తో కింద తెలిపిన విధంగా పూరించాలి.
   ఉదాహరణ : Ⓐ Ⓑ ● Ⓓ
   (C) సరైన ప్రతిస్పందన అయితే
5. ప్రశ్నలకు ప్రతిస్పందనలను ఈ ప్రశ్నపత్రముతో ఇవ్వబడిన OMR పత్రము పైన ఇవ్వబడిన వృత్తాల్లోనే పూరించి గుర్తించాలి. అలాకాక సమాధాన పత్రంపై వేరొక చోట గుర్తిస్తే మీ ప్రతిస్పందన మూల్యాంకనం చేయబడదు.
6. ప్రశ్న పత్రము లోపల ఇచ్చిన సూచనలను జాగ్రత్తగా చదవండి.
7. చిత్తుపనిని ప్రశ్నపత్రము చివర ఇచ్చిన ఖాళీస్థలములో చేయాలి.
8. OMR పత్రము పై నిర్దీత స్థలంలో సూచించవలసిన వివరాలు తప్పించి ఇతర స్థలంలో మీ గుర్తింపును తెలిపే విధంగా మీ పేరు రాయడం గానీ లేదా ఇతర చిహ్నాలను పెట్టడం గానీ చేసినట్లయితే మీ అనర్హతకు మీరే బాధ్యులవుతారు.
9. పరీక్ష పూర్తయిన తర్వాత మీ OMR పత్రాన్ని తప్పనిసరిగా పరీక్ష పర్యవేక్షకుడికి ఇవ్వాలి. వాటిని పరీక్ష గది బయటకు తీసుకువెళ్లకూడదు. పరీక్ష పూర్తయిన తరువాత అభ్యర్థులు ప్రశ్న పత్రాన్ని, OMR పత్రం యొక్క కార్బన్ కాపీని తీసుకువెళ్లవచ్చు.
10. నీలి/నల్ల రంగు బాల్ పాయింట్ పెన్ మాత్రమే ఉపయోగించాలి.
11. లాగరిథమ్ టేబుల్స్, క్యాలిక్యులేటర్‌లు, ఎలక్ట్రానిక్ పరికరాలు మొదలగునవి పరీక్షగదిలో ఉపయోగించడం నిషేధం.
12. తప్పు సమాధానాలకు మార్కుల తగ్గింపు లేదు.

MATHEMATICAL SCIENCES

Paper - III

1. Suppose $x \in Q$, the set of all rational numbers, with $x=\frac{p}{q}$ where p, q $\in$ Z and q > 0. Define $f : \mathbf{R} \to \mathbf{R}$ by :

$$f(x)=\begin{cases} 1 & \text{if } x=0 \\ 0 & \text{if } x \in \mathbf{R}-Q \\ \frac{1}{q} & \text{if } x \in Q \end{cases}$$

The set of continuities of f in **R** is :

(A) ϕ, the empty set

(B) Q

(C) $\mathbf{R}-Q$

(D) **R**

2. Let α and β respectively denote the limit inferior and limit superior of $\{S_n\}$, where $S_n=(-1)^n\left(1+\frac{1}{n}\right)$ for n = 1, 2, 3, ... then $\{S_n : \alpha \leq S_n \leq \beta\}=$

(A) $\{S_n : n=1, 2, 3, ...\}$

(B) **R**

(C) ϕ

(D) $\left\{\frac{n+1}{n} : n \in \mathbf{Z}, n \neq 0\right\}$

3. Let $\mathbf{R}^2$ be the Euclidean plane with usual metric and let

$$E=\left\{\left(\frac{1}{n}, 1-\frac{1}{n}\right) : n=1, 2, 3,.....\right\} \subseteq \mathbf{R}^2.$$

The interior of E in $\mathbf{R}^2$ is :

(A) $\{(0, 1)\}$

(B) ϕ, the empty set

(C) E

(D) $\mathbf{R}^2$

4. (a) A function $f:(a, b) \to \mathbf{R}$ is uniformly continuous as (a, b) if and only if there is a continuous extension $\overline{f}$ of f on [a, b].

(b) The function $f : \left(0, \frac{1}{\pi}\right] \to \mathbf{R}$ defined by $f(x)=x \sin\left(\frac{1}{x}\right)$ is uniformly continuous on $\left(0, \frac{1}{\pi}\right]$

(A) (a) is true, (b) is false

(B) (a) is false, (b) is true

(C) (a) and (b) are both true; and (b) follows from (a)

(D) (a) and (b) are both true but (b) does not follow from (a)

5. In **R**, let **C** be the set of all closed sets, **B** is the set of all Bold sets and **M** be the set of all Lebesgue measurable sets then :

(A) $M \subseteq B \subseteq C$ (B) $C \subseteq B \subseteq M$

(C) $M \subseteq C \subseteq B$ (D) $B \subseteq M \subseteq C$

6. Let $f : [a, b] \to \mathbf{R}$ be a bounded function, and let $P = \{x_0, x_1, x_2, \ldots, x_n\}$ be a partition of [a, b]. If m_i and M_i respectively denote the infimum and supremum of $\{f(x) : x \in [x_{i-1}, x_i]\}$ and $Q(x)$ is the step function taking the values m_i for $x \in [x_{i-1}, x_i]$ for $i = 1, 2, \ldots, n$, then the Lebesgue integral of **Q** over [a, b] is :

(A) $\underline{\int_a^b} f(x)\,dx$

(B) $\overline{\int_a^b} f(x)\,dx$

(C) L (P, f)

(D) U (P, f)

7. Let (X, d) be a metric space and $G_i = \left\{x \in X : d(x, x_i) < \frac{1}{n}\right\}$ for $i = 1, 2, 3, \ldots$ Then $\bigcap_{i=1}^{\infty} G_i$ is :

(A) open in X

(B) not necessarily open in X

(C) equal to the empty set

(D) equal to X

8. If $E \subseteq \mathbf{R}$ has the property " $a \in E$, $c \in E$ and $a < b < c$ implies $b \in E$" then E is :

(A) closed (B) dense

(C) connected (D) compact

9. If R is the radius of convergence of the power series $\sum_{n=0}^{\infty} a_n x^n$ then the radius of convergence of $\sum_{n=0}^{\infty} \frac{a_n}{n+1} x^n$ is :

(A) 0 (B) R

(C) $\frac{1}{R}$ (D) 2R

10. Which of the following is a sub space of the vector space $\mathbf{R}^2$ over the field **R**.

(A) $\{(a, b) \in \mathbf{R} \times \mathbf{R} / a = 3b\}$

(B) $\{(a, b) \in \mathbf{R} \times \mathbf{R} / a + 2b + 2 = 0\}$

(C) $\{(a, b) \in \mathbf{R} \times \mathbf{R} / a^2 = b^2\}$

(D) $\{(a, b) \in \mathbf{R} \times \mathbf{R} / ab = 0\}$

11. Let $B=\{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ be a basis of the vector space $\mathbf{R}^3(\mathbf{R})$ and $T : \mathbf{R}^3 \to \mathbf{R}^3$ be the Linear transformation defined by :

$T(x, y, z) = (x+2y, y-z, z)$ for all $(x, y, z) \in \mathbf{R}^3$. Then the matrix $[T]_B =$

(A) $\begin{pmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 3 & 1 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & -1 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

12. The matrix

$A = \begin{bmatrix} 2i & 3+4i & 2+i \\ 4i-3 & 0 & 5 \\ i-2 & -5 & -7i \end{bmatrix}$ is a :

(A) Symmetric matrix

(B) Skew symmetric matrix

(C) Hermitian matrix

(D) Skew Hermitian matrix

13. If the system of equations

$x+2y+3z=5$

$2x+3y+z=-3$

$3x+2y+\lambda z=\mu$

has infinitely many solutions, then $2\lambda-\mu=$

(A) 15 (B) 59

(C) -15 (D) -59

14. The sum of Eigen values of the matrix $\begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ is :

(A) 5 (B) 4

(C) 6 (D) 7

15. If u, v are elements of an inner product vector space V (C), then

$\sum_{n=0}^{3} i^n \|u+i^n(v)\|^2 =$

(A) $i < u, v >$

(B) $4i < u, v >$

(C) $< u, v >$

(D) $4 < u, v >$

16. Suppose C is the circle $|z|=2$ positively oriented. Then, $\int_C \frac{1}{z^2 \sinh z} dz =$

(A) $\frac{-\pi i}{3}$ (B) $\frac{\pi i}{3}$

(C) $\frac{2\pi i}{3}$ (D) $\frac{-2\pi i}{3}$

17. Let C be the positively oriented unit circle $|z|=1$. Then $\int_C z \sec z \, dz =$

(A) $2\pi i$ (B) $-2\pi i$

(C) 0 (D) πi

18. $\int_0^{2\pi} e^{e^{it}-4it} dt =$

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{12}$

(C) $\frac{\pi}{24}$ (D) $\frac{\pi}{8}$

19. Let C : $|z-1|=\frac{3}{2}$ be the circle positively oriented. Then $\int_C \cot \pi z \, dz =$

(A) i (B) 2i

(C) πi (D) $2\pi i$

20. Let C be the unit circle $|z|=1$ positively oriented. Then

$$\int_C \frac{7z^6-15z^2+1}{z^7-5z^3+z-1} dz =$$

(A) $4\pi i$ (B) $5\pi i$

(C) $6\pi i$ (D) $7\pi i$

21. Suppose $f(z)=\frac{\sinh z}{z^6}$. Then residue of $f(z)$ at $z=0$ is :

(A) 60 (B) $\frac{1}{60}$

(C) 120 (D) $\frac{1}{120}$

22. Suppose $z=x+iy$, $|z|=r$ and $\text{Arg} z=\theta$. Then, real part of principal value of z^z is $e^A \cos(y \ln r + \theta x)$, where A =

(A) $x \ln r - y\theta$ (B) $x \ln r + y\theta$

(C) $y \ln r - x\theta$ (D) $x \ln r + x\theta$

23. The number of generators of a cyclic group of order 4900 is :

(A) 420 (B) 840

(C) 1680 (D) 1260

24. If a set A has 4 elements, then the number of binary operations that can be defined on A is :

(A) 2^{32} (B) 2^{16}

(C) 2^{8} (D) 2^{4}

25. Let Z_{11} be the set of all residue classes of integers modulo 11, $Z_{11}^{*} = Z_{11} - \{\bar{0}\}$ and X_{11} denote the multiplication modulo 11. Then in the group $\left(Z_{11}^{*}, X_{11}\right)$,

$\bar{4}\ X_{11} (\bar{5})^{-1} =$

(A) $\bar{9}$ (B) $\bar{7}$

(C) $\bar{8}$ (D) $\bar{3}$

26. A prime ideal in the ring $(Z, +, \cdot)$ of all integers which is not a maximal ideal among the following is :

(A) 2**Z** (B) 3**Z**

(C) 5**Z** (D) {0}

27. If Z_n denote the set of all residue classes of integers modulo n, then the number of non-zero nil potent elements in the ring $(Z_{16}, +_{16}, \times_{16})$ is :

(A) 1 (B) 3

(C) 5 (D) 7

28. If Z_n denote the set of all residue classes of integers modulo n, then the number of zero divisions in the ring $(Z_{24}, +_{24}, \times_{24})$, other than $\bar{0}$, is :

(A) 3 (B) 4

(C) 6 (D) 8

29. Let $X = \{1, 2, 3, 4\}$ and $\tau = \{\phi, X, \{1, 2\}, \{3, 4\}\}$. If $A = \{2, 3\}$, Then, in the topological space (X, τ), A is :

(A) a dense subset

(B) a nowhere dense subset

(C) an open set

(D) a closed set

30. In the metric space **R** with Euclidean metric, a compact set among the following is :

(A) (4, 5) (B) [7, 8)

(C) [2, 3] (D) (5, 6]

31. Eliminating y from the system

$$\frac{dx}{dt}=x(1-y)$$

$$\frac{dy}{dt}=-y(1-x)$$

We obtain the nonlinear second order equation satisfied by the function $x(t)$ as :

(A) $\frac{d^2x}{dt^2}=x(x-1)\frac{dx}{dt}+x^2(1-x)+\left(\frac{dx}{dt}\right)^3$

(B) $x\frac{d^2x}{dt^2}=x(x-1)\frac{dx}{dt}+x^2(1-x)+\left(\frac{dx}{dt}\right)^2$

(C) $x^2\frac{d^2x}{dt^2}=x(1-x)\frac{dx}{dt}+x(1-x^2)+\left(\frac{dx}{dt}\right)^2$

(D) $\frac{d^2x}{dt^2}=x(x-1)\frac{dx}{dt}+x^2(1-x)+\left(\frac{dx}{dt}\right)^2$

32. All the numbers λ for which the boundary value problem $y''+\lambda y=0$, $y(0)=0$ and $y(1)=0$ has a nontrivial solution are :

(A) $\lambda=n\pi$, $n=1, 2, ...$

(B) $\lambda=n^2\pi^2$, $n=1, 2, ...$

(C) $\lambda=n^3\pi^2$, $n=1, 2, ...$

(D) $\lambda=n\pi^2$, $n=1, 2, ...$

33. The particular solution y of $y''-2y'-3y=64\,xe^{-x}$ is :

(A) $y(x)=e^{x}(8x^2+4x-1)$

(B) $y(x)=e^{-x}(8x^2-4x+1)$

(C) $y(x)=e^{-2x}(8x^2-4x-1)$

(D) $y(x)=-e^{-x}(8x^2+4x+1)$

34. The Green's function $G(x, s)$ that is associated with the general solution of the boundary value problem

$-y''=f(x)$, $y(0)=0$, $y(1)=0$ is :

(A) $G(x, s)=\begin{cases} x(1-s), & 0\leq s\leq x \\ s(x-1), & x\leq s\leq 1 \end{cases}$

(B) $G(x, s)=\begin{cases} s(1-x), & 0\leq s\leq x \\ x(1-s), & x\leq s\leq 1 \end{cases}$

(C) $G(x, s)=\begin{cases} x(1-s^2), & 0\leq s\leq x \\ s(1-x), & x\leq s\leq 1 \end{cases}$

(D) $G(x, s)=\begin{cases} s^2(1-x), & 0\leq s\leq x \\ x(1-s^2), & x\leq s\leq 1 \end{cases}$

35. A solution of :

$\frac{\partial^2 u}{\partial x^2}-6\frac{\partial^2 u}{\partial x\partial y}+9\frac{\partial^2 u}{\partial y^2}=0$ is :

(A) $u=f(y+3x)+g(y+3x)$

(B) $u=f(y+3x)+xg(y+3x)$

(C) $u=f(y+4x)+g(y+2x)$

(D) $u=f(y+x)+g(y+2x)$

36. The region in which, the equation

$$(x^2-1)\frac{d^2u}{dx^2}+2y\frac{\partial^2 u}{\partial x \partial y}-\frac{\partial^2 u}{\partial y^2}=0$$

is elliptic is :

(A) $\{(x, y) : x^2+y^2 > 1\}$

(B) $\{(x, y) : x^2+y^2 = 1\}$

(C) $\{(x, y) : x^2+y^2 < 1\}$

(D) $\mathbf{R}\times\mathbf{R}$

37. The general solution of the equation :

$$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}=x-y \text{ is :}$$

(A) $z=2x(x-y)+Q_1(x+y)+Q_2(x-y)$

(B) $z=2x^2(x-y)^2+Q_1(x+y)+Q_2(x-y)$

(C) $z=\frac{1}{2}x(x-y)+Q_1(x+y)+Q_2(x-y)$

(D) $z=\frac{1}{4}x(x-y)^2+Q_1(x+y)+Q_2(x+y)$

38. The solution of the two - dimensional Laplace equation :

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$$

satisfying the boundary conditions $u(0, y)=0,\ 0 \leq y \leq b$

$u(a, y)=0,\ 0 \leq y \leq b$

and $u(x, 0)=0,\ 0 \leq x \leq a$ is

(A) $u(x, y)=\sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{a}\cosh\frac{n\pi y}{a}$

(B) $u(x, y)=\sum_{n=1}^{\infty} c_n \cos\frac{n\pi x}{a}\cosh\frac{n\pi y}{a}$

(C) $u(x, y)=\sum_{n=1}^{\infty} c_n \sin\frac{n\pi x}{a}\sinh\frac{n\pi y}{a}$

(D) $u(x, y)=\sum_{n=1}^{\infty} c_n \cos\frac{n\pi x}{a}\sinh\frac{n\pi y}{a}$

39. In the Gauss elimination method, to solve a linear system of n equations in n unknowns, the number $f(n)$ of operations required is :

(A) 0(n)

(B) $0(n^2)$

(C) $0(n^3)$

(D) $0(n^4)$

40. Suppose $f(x)$ is continuous on the interval [a, b] and that $f(a)\,f(b) < 0$. If p is a zero of $f(x)$ in [a, b] and if $\{p_n\}$ is a sequence approximating p, then $|p_n - p| \leq$:

(A) $\frac{b+a}{2^n}, n \geq 1$

(B) $\frac{b-a}{2^{n-1}}, n \geq 1$

(C) $\frac{b-a}{2^{n+1}}, n \geq 1$

(D) $\frac{b-a}{2^n}, n \geq 1$

41. The extremal of the functional $\int_a^b (y + y'^2 + 2ye^x)\,dx$ is :

(A) $y = \frac{1}{2} xe^x + C_1 e^x + C_2 e^{-x}$

(B) $y = Cxe^x$

(C) $y = C_1 e^x + C_2 e^{-x}$

(D) $y = (C_1 + C_2 x)e^x$

42. The shape that maximizes the area enclosed by a rectangle of given perimeter p is a :

(A) parallelogram

(B) rhombus

(C) trapezium

(D) square

43. If $K(x, t) = \begin{cases} x(1-t) \text{ if } 0 \leq x \leq t \leq 1 \\ t(1-x) \text{ if } 0 \leq t \leq x \leq 1 \end{cases}$ then the eigen functions of $f(x) = \lambda \int_0^1 K(x, t)\, f(t)\, dt$ are:

(A) sin mπx, m∈N

(B) cos mπx, m∈N

(C) sinh mπx, m∈N

(D) cosh mπx, m∈N

44. The nonzero eigen value of the linear integral equation $f(x) = \lambda \int_0^1 e^x e^t f(t)\, dt$ is :

(A) $\frac{2}{e+1}$

(B) $\frac{2}{e^2-1}$

(C) $\frac{2}{e^2+1}$

(D) $\frac{2}{e-1}$

45. A mass of 2 kg is thrown horizontally due north with velocity 5 km per second in latitude 45° N. The magnitude of Coriolis force acting on the mass is :

(A) 0.2576 N

(B) 0.5142 N

(C) 1.0284 N

(D) 2.0568 N

46. A fair coin is tossed repeatedly unless a head is obtained. The probability that the coin has to be tossed at least four times is :

(A) $\frac{1}{2}$

(B) $\frac{1}{8}$

(C) $\frac{1}{4}$

(D) $\frac{1}{6}$

47. The function $f(x) = k \cdot x\,(1-x)$, $0 < x < 1$ defines a p.d.f, if k is :

(A) $\frac{1}{6}$

(B) $\frac{1}{3}$

(C) $\frac{1}{2}$

(D) 6

48. Let X be a random variable with p.d.f $f(x) = \frac{1}{2} \cdot \exp\left(-\frac{x}{2}\right)$, $x > 0$. Then moment generating function of X is :

(A) $\frac{1}{1-2t}$

(B) $\frac{1}{1-2t}$, $t < \frac{1}{2}$

(C) $\frac{1}{(1-2t)^2}$

(D) $\frac{2}{1-2t}$, $t < \frac{1}{2}$

49. $X_n \xrightarrow{P} 1 \Rightarrow$

(A) $\frac{1}{X_n} \xrightarrow{L} 1$

(B) $\frac{1}{X_n} \xrightarrow{a \cdot s} 1$

(C) $\frac{1}{X_n} \xrightarrow{P} 1$

(D) Both (A) and (B)

50. Let X be a random variable with p.d.f. $f(x) = 1, 0 < x < 1$. Then

$P\left\{\left|X - \frac{1}{2}\right| \le 2\sqrt{\frac{1}{12}}\right\}$ is :

(A) $\frac{45}{49}$

(B) $\frac{1}{12}$

(C) $\frac{1}{2}$

(D) $\frac{1}{80}$

51. The transition matrix of a Markov chain is given by $P = \begin{matrix} & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{matrix}$. Then the chain is :

(A) reducible

(B) ergodic

(C) not absorbing

(D) regular

52. Let X(t) be a Poisson process with parameter λ. Then $E[\{X(t) - X(s)\}^2]$ for $t > s$ is :

(A) $\lambda t(1 + \lambda t)$

(B) $\lambda(t - s)$

(C) $\lambda^2 t^2$

(D) $\lambda(t - s)\ [1 + \lambda(t - s)]$

53. Let $X_{(1)}, X_{(2)}, X_{(3)}$ be the order statistics of I'.I'.d random variables X_1, X_2, X_3 with common p.d.f.

$f(x) = \beta\, e^{-x\beta}, x > 0, \beta > 0$

The joint p.d.f of $X_{(1)}, X_{(2)}, X_{(3)}$ is :

(A) $3 \cdot \beta^3(-\beta y_1 - \beta y_2 - \beta y_3)$, $y_1 < y_2 < y_3$

(B) $\beta^3 \exp(-\beta y_1 - \beta y_2 - \beta y_3)$, $y_1 < y_2 < y_3$

(C) $3! \exp(-\beta y_1 - \beta y_2 - \beta y_3)$, $y_1 < y_2 < y_3$

(D) $3!\ \beta^3 \exp(-\beta y_1 - \beta y_2 - \beta y_3)$, $y_1 < y_2 < y_3$

54. A man rolls a fair die again and again until he obtains 4 or 5. The probability that he will require 4 throws is :

(A) $2^3/3^5$

(B) $2/3^4$

(C) $2^3/3^4$

(D) $2^3/3^2$

55. Let X~N(1, 2), Y~N(1.5, 1) be two independent random variables. Then X+Y is distributed as :

(A) N (3.5, 2) (B) N (2.5, 3)

(C) N (2.5, 2) (D) N (3.5, 1)

56. If a binomial random variable has mean = 4 and variance = 3, then its third central moment is :

(A) $\frac{1}{2}$

(B) $\frac{5}{2}$

(C) $\frac{3}{2}$

(D) $\frac{7}{4}$

57. Let $X_1, X_2 ...... X_n$ be I'.I'.d random variables with $E|X_1|^K < \infty$ for some positive integer K. Then $\sum_{j=1}^{n} X_j^K/n \xrightarrow{p}$

(A) EX_1 as $n \to \infty$

(B) EX_1^2 as $n \to \infty$

(C) $E(X_1^K)$ as $n \to \infty$

(D) $V(X_1^K)$ as $n \to \infty$

58. In sampling from a normal population $\overline{X}^2$ is sufficient for :

(A) μ^2

(B) μ

(C) $\sigma^2 + \mu^2$

(D) None of these

59. Any size of likelihood ratio test is attainable if the distribution function of the likelihood ratio is :

(A) degenerate

(B) absolutely continuous

(C) decreasing

(D) no jump points

60. The maximum likelihood estimator is generally :

(A) consistent

(B) sufficient

(C) unbiased

(D) unique

61. In sampling from N (μ, σ). The critical region for testing $H_0 = \mu = \underset{0}{\mu}$ against $H_1 : = \underset{0}{\mu} < \underset{1}{\mu}$ is of the form :

(A) $\bar{x} < \lambda_2$

(B) $\bar{x} > \lambda_1$

(C) $\bar{x} < \underset{0}{\mu} + \frac{\sigma}{\sqrt{n}}$

(D) $\rho\left(\bar{x} < \lambda_2 / H_0\right) = \alpha$

62. The distribution of likelihood ratio in large samples is :

(A) Normal

(B) Chi-square

(C) t^2

(D) Standard normal

63. The pivot is a function of :

(A) unbiased statistic

(B) consistent statistic

(C) sufficient statistic and parameter

(D) sufficient statistic

64. This test ignores the magnitude of the difference between the observations and the hypothesized quantile.

(A) wilcoxon - signed rank test

(B) run test

(C) kolmogorov - smirnov test

(D) sign test

65. Two jointly multinormal vectors are independent :

(A) if they are uncorrelated

(B) if and only if they are uncorrelated

(C) if the covariance matrices are positive definite

(D) the individual elements each have univariate normal

66. If M~Wp (Σ, m) and B is a ($p \times q$) matrix, then B′MB~ :

(A) Wp ($B'\Sigma B$, m)

(B) Wp (I, m)

(C) Wq ($B'\Sigma B$, m)

(D) Wq (I, m)

67. The normal equations in the linear model $\underset{(k\times 1)}{Y = X\beta + e}$ are :

(A) $X'X\beta = XY$

(B) $(X'X) = X'Y$

(C) $X'Y\beta = X'Y$

(D) $X'X\beta = X'Y$

68. In the simple linear regression model the distribution of the test statistic for testing the significance of the independent variable is :

(A) $F(2, n-2)$

(B) t_{n-2}

(C) t^2_{n-2}

(D) Standard normal

69. The finite population correction is given by the quantity :

(A) $\frac{n}{N}$

(B) $\frac{N-n}{N}$

(C) $\sqrt{\frac{N-n}{N}}$

(D) $\sqrt{\frac{n}{N}}$

70. This allocation gives a self - weighing sample :

(A) optimum

(B) probability proportional to size

(C) proportional

(D) equal

71. A systematic sample does not yield good results if :

(A) variations in units is periodic

(B) units at regular intervals are correlated

(C) both (A) and (B)

(D) None of (A) and (B)

72. The relationship between plot size x and plot variance V_x is given by, in the usual notation :

(A) $\log x = \log V_1 - b' \log V_x$

(B) $\log x = \log V_x - b' \log V_1$

(C) $\log V_x = \log x - b' \log V_1$

(D) $\log V_x = \log V_1 - b' \log x$

73. The error mean square for RBD was 60 percent to that of CRD error mean square was due to :

(A) Fairfield Smith

(B) Cox

(C) Fisher

(D) Cochran

74. The failure distribution with a constant hazard rate is :

(A) Weibull

(B) Exponential

(C) Log-normal

(D) Gamma

75. The mean Waiting time formula in M|G|1 was due to :

(A) Harris

(B) Gauss

(C) Pollaczek - Khinchin

(D) Erlang

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Space For Rough Work

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